RESEARCH STATEMENT

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1. Overview

My research interests lie in number theory and representation theory, with a particular focus on the Langlands program.

The Langlands program proposes a far-reaching unification of arithmetic and representation theory through a network of deep conjectures. It traces back to a letter Langlands wrote to Weil in 1967, where he envisioned a vast generalization of class field theory that would connect Galois representations with automorphic forms on reductive groups. While class field theory exemplifies the abelian case for GL_1, the modularity theorem, central to the proof of Fermat's Last Theorem, realizes the case of GL_2.

The Langlands program remains largely conjectural in the global case. However, substantial progress has been achieved in the local theory over the past decades. In many cases, local correspondences for a reductive group G have been established—most notably when G is GL_n or a classical group—where irreducible admissible representations are parametrized by L-parameters, namely certain representations of the Weil–Deligne group WD_F of the base field F into the L-group ${}^LG(\mathbb{C})$ of G. In [BX25], we computed the Rankin–Selberg gamma factor for a simple supercuspidal representation of U_{2l} twisted by a multiplicative character. This computation contributes to the explicit realization of the local Langlands correspondence for the quasi-split unitary group U_{2l}.

Another aspect of the Langlands program is the relative Langlands program. The philosophy is, for a spherical subgroup H of G, the H-periods of cusp forms of G, or more generally representation-theoretic functionals on G, are intimately connected with L-functions. This theory has deep historical roots, ranging from the Waldspurger formula to the Gan–Gross–Prasad conjectures. Significant progress has been made in recent years, particularly in the local setting. In [Xin25], we studied the Bessel functional, one of the central objects in the Gan–Gross–Prasad framework, over a nonarchimedean local field. In [WX24], we considered the pair $(G, H) = (\mathrm{U}_{2n}, \mathrm{Sp}_{2n})$ over a number field and studied the H-distinguished spectrum.

2. Unramified Bessel function for GSpin_{2n+1} [Xin25]

A model provides a realization of an automorphic representation through certain equivariant functionals, namely, Fourier coefficients obtained by integrating along a specific subgroup. In the representation theory of orthogonal and general spin groups, the Bessel models play a central role in constructing automorphic L-functions for GSpin $\times$ GL via the Rankin–Selberg method. These models are defined by Bessel functionals, that is, periods over Bessel subgroups giving rise to equivariant functions.

The concept also has a local analogue. In this work, we study the local Bessel model for unramified representations and obtain an explicit formula for the unramified Bessel functions of odd general spin groups, thereby extending the results known for orthogonal groups in [BFF97]. Such formulas in general are needed in local computation of global construction. In particular, this one has an application in the local unramified computation in the global Rankin–Selberg integral [ACS24].

Let F be a nonarchimedean local field of characteristic zero and let G be the split reductive group $\mathrm{GSpin}_{2n+1}(F)$ $(n \geq 1)$ over F. Let $P = M \ltimes U$ be the standard parabolic subgroup whose Levi part M is $\mathrm{GL}_1^{n-1} \times \mathrm{GSpin}_3$. Here U is the unipotent radical of P. Let θ be a non-degenerate character on U and T be the identity component of the stabilizer of θ in M. We consider the Bessel model with respect to the Bessel subgroup $R = T \ltimes U$. (There are Bessel subgroups with a higher rank reductive part and lower dimensional unipotent part. Due to certain reductive arguments, it is only interesting to consider the computation for the minimal rank case, which we considered here.)

A Bessel functional of a representation π is an equivariant functional, i.e., a functional lies in the space

$$\mathrm{Hom}_R(\pi, \theta \otimes \lambda)$$

where λ is a character of T. Bessel functionals are unique up to scalar multiplication, that is, the above space is of zero or one dimension. In this work, by writing an R-period integral B and showing its convergence, we defined a Bessel functional. In line with the orthogonal group, we also established its meromorphic continuation and functional equation.

The main object we consider is the normalized unramified function $\mathcal{H}_\chi$ in the Bessel model of an unramified principal series $\pi = \mathrm{Ind}\chi$. It is defined as

$$\mathcal{H}_\chi(g) = (*) \times B(\pi(g)\phi_{K,\chi}),$$

where $\phi_{K,\chi}$ is the standard spherical vector in the unramified principal series and $(*)$ is a normalizing factor so that $\mathcal{H}_\chi$ on K is the identity. This is the analogue of the normalized unramified Whittaker function.

Let $\chi = (\chi_0, \ldots, \chi_n)$ be the unramified character, with the inducing unramified characters χ_i parametrized by non-zero complex numbers $\alpha_i = \chi_i(\varpi)$, where ϖ is a fixed uniformizer in F. The main result we obtained in this paper is a Casselman–Shalika type formula. We explain this in the non-split case, i.e., when T is isomorphic to the non-split GSpin_2. By the equivalence properties the function satisfies and by some short computations, the unramified Bessel function is indeed given by its evaluations on a set of representatives of the double cosets $R\backslash G/K$, where we choose certain torus elements ϖ^δ indexed by integral dominant weights $\delta = (\ell_1, \ldots, \ell_n)$. The image of such an element in SO_{2n+1} under the fixed projection is the diagonal element $\mathrm{diag}(\varpi^{\ell_1}, \ldots, \varpi^{\ell_n})$.

Theorem 2.1. *Suppose T is non-split, the Bessel function is given by*

$$H_\chi(\varpi^\delta) = \frac{\delta_B(\varpi^\delta)}{1-q^{-1}} \times \frac{\mathcal{A}\left(\prod_{i=1}^n \alpha_i^{\ell_i+i}(1-\alpha_0^{1/2}\alpha_i^{-1}q^{-\frac{1}{2}})(1+\alpha_0^{1/2}\alpha_i^{-1}q^{-\frac{1}{2}})\right)}{\mathcal{A}(\alpha_1^n \ldots \alpha_n)},$$

where $\mathcal{A} = \sum_{w\in W}(-1)^{length(w)}w$ is the alternator in the group algebra $\mathbb{C}[W]$ and δ_B is the modulus character of the Borel subgroup B.

We also obtained the split case, with a similar formula. In either case, the formula is obtained by using the Casselman–Shalika method.

In the last section, we construct a Rankin–Selberg integral for cuspidal automorphic representations of $\mathrm{GSpin}_{2n+1} \times \mathrm{GL}_n$. As an application of the explicit formula we obtain, we compute its local factor at a good place.

Theorem 2.2. *Suppose the global Bessel period does not vanish. Let π and σ be cuspidal representations of GSpin_{2n+1} and GL_n, respectively. The Rankin–Selberg integral we define for $\pi \otimes \sigma$ is factorizable.*

Moreover, at a good place v, the local factor equals

$$L(s+1/2,\, \pi_v \times \sigma_v).$$

It is worth mentioning that a corollary of the proof of the above theorem contributes to the recent construction of a Rankin–Selberg integral for GSpin $\times$ GL by Asgari, Cogdell, and Shahidi [ACS24], particularly to their unramified computation.

3. Gamma factor of a simple supercuspidal representation for U_{2n} [BX25]

The local Langlands correspondence for a quasi-split reductive group G over a p-adic field F asserts a finite-to-one surjective map

$$\mathrm{LLC}_G : \Pi(G) \longrightarrow \Phi(G),$$

from the set of equivalence classes of irreducible smooth representations of $G(F)$ to the set of L-parameters.

This correspondence was first established for general linear groups GL_n by Henniart and by Harris–Taylor, building on earlier works of Langlands, Carayol, and others. For classical groups, the approach proceeds via the principle of *functoriality.* Arthur proved the correspondence for symplectic and orthogonal groups by developing the theory of endoscopic classification [Art13]. Following his approach, Mok subsequently established the result for quasi-split unitary groups [Mok15]. Given this background, a natural question to ask is:

Can one describe the endoscopic lift explicitly, even for particular representations?

This problem is highly nontrivial. Nevertheless, significant progress has been made in the case where π is a *simple supercuspidal* representation. Such representations are defined to be supercuspidal representations of minimal positive depth, which turn out to be constructed via compact induction from an affine generic character. The study of simple supercuspidal representations provides important insights and serves as a testing ground for understanding more general supercuspidal representations.

When the quasi-split group G is the unitary group U_N with respect to an unramified quadratic extension E/F, and the residue characteristic of F is not 2, Oi [Oi19] established a beautiful result: the endoscopic lift Π, a representation of $\mathrm{GL}_n(E)$, of a simple supercuspidal representation π of $\mathrm{U}_N(F)$ is again simple supercuspidal using character formulas. Moreover, an explicit description of the correspondence through their inducing data was given. However, the restriction to the non-dyadic case cannot be removed in his proof. In general, so far the only known method to address the dyadic case is via the study of local factors of representations.

The method has proven highly effective for orthogonal groups SO_N [AHKO25]. In that work, the authors computed the local gamma factors. By analyzing the poles at $s = 1$, and by combining this with structural results on the L-parameters, for simple supercuspidal π of SO_N, they obtained an explicit description of the endoscopic lift to GL_N and the associated Langlands parameter.

In [BX25], we analyzed the local gamma factors for simple supercuspidal representations $\pi \times \tau$ of $\mathrm{U}_{2l} \times \mathrm{GL}_1$ through the local Rankin–Selberg integrals. Fix a level-one additive character of F, and a tamely ramified character Υ extending $\omega_{E/F}$, the quadratic character associated with the field extension E/F in the class field theory. The Rankin–Selberg integral $\mathcal{L}(W, f_s, \phi)$ has the input data of a Whittaker function $W \in (\mathcal{W}, \psi_{N_l}^{-1})$, a section in the parabolic induction $f_s \in V(\tau, s)$ and a Schwartz function

$\phi \in \mathcal{S}(E)$. The gamma factor is then defined as

$$\gamma(s, \pi \times \tau, \Upsilon, \psi) = \omega_\pi(-1)\tau(-1)^l C(s, \tau, \psi) \frac{\mathcal{L}(W, M(\tau, s)f_s, \phi)}{\mathcal{L}(W, f_s, \phi)}$$

where $C(s, \tau, \psi)$ is the local coefficient (which we show to equal a Tate's gamma factor) and $M(\tau, s)$ is the standard intertwining operator. The gamma factors defined above are known to coincide with the local gamma factors defined by the Langlands–Shahidi method [Mor25].

We note that the gamma factors are independent of the choice of the Whittaker function W and so on, while they are only dependent on the representations. By finding a proper choice of the data of a Whittaker function W, a section f_s and a Schwartz function ϕ, we obtained the two local integrals in the definition of the gamma factor explicitly and consequently an explicit formula for the gamma factor.

Theorem 3.1. *Let $\pi = \pi_{\omega^1, b}$ be a simple supercuspidal representation (indexed by a character ω^1 of the residue field κ_F and $b \in \mathcal{O}_F^\times$), τ be a character of GL_1 and ψ be a level-one additive character. Then*

$$\gamma(s, \pi \times \tau, \Upsilon, \psi) = \omega^1(-1)\tau(-1)^l q_F^{-2s+1} \tau(b^{-1}\varpi).$$

Equivalently, the gamma factor defined in the Langlands–Shahidi method is given by

$$\gamma^{sh}(s, \pi \times \tau, \psi) = -\omega^1(-1)\tau(-1)^l \tau(b^{-1}\varpi) q_F^{-2s+1},$$

According to the formula, the γ-factor has no pole at $s = 1$. In line with the results for odd orthogonal groups [Adr16, AHKO25], it is reasonable to expect that the endoscopic lift remains simple supercuspidal, and that the explicit correspondence formally extends Oi's result for the non-dyadic cases [Oi19].

4. Sp_{2n}-Distinguished U_{2n} spectrum [WX24]

Let F be a number field, and $\mathbb{A}$ be the ring of adeles. Denote the adelic quotient as $[G] = G(F)\backslash G(\mathbb{A})$. For a closed subgroup H of G, in the theory of automorphic forms one is often interested in the period integral

$$\mathcal{P}_H(\varphi) = \int_{[H]} \varphi(h) dh.$$

An automorphic representation π of G is H-*distinguished* if $\mathcal{P}_H$ defines a nontrivial functional on V_π. Distinguished representations are closely related to functoriality and special values of L-functions.

The Langlands spectral decomposition is loosely written as

$$L^2([G]) = \underbrace{L^2_{\text{cuspidal}}([G]) \oplus L^2_{\text{residual}}([G])}_{\text{discrete spectrum}} \oplus \underbrace{L^2_{\text{Eisenstein}}([G])}_{\text{continuous spectrum}} .$$

In this work, we considered the pair $(G, H) = (\mathrm{U}_{2n}, \mathrm{Sp}_{2n})$ and studied its H-distinguished spectrum. In particular, we provided an upper bound for the H-distinguished spectrum. The upper bound is an intermediate step of constructing a nontrivial discrete distinguished spectrum.

The H-distinguished spectrum in the cuspidal subspace $L^2_0([G])$ is defined to be the space of those with nonvanishing period integrals. While the convergence of a period integral of a cusp form is guaranteed in general, there is no such theorem for a general automorphic form. As a consequence, we study the period integral of pseudo-Eisenstein series and get the spectral information in the distributional sense. Let the H-distinguished spectrum be denoted by $L^2_{H-\text{dist}}([G])$, essentially as the orthogonal complement to the space of pseudo-Eisenstein series with a vanishing period integral. We obtained an upper bound with respect to the residue data:

$$L^2_{H-\mathrm{dist}}([G]) \subseteq \hat{\bigoplus_{(\mathfrak{X},\mathfrak{C})\in\mathfrak{B}_{H-\mathrm{dist}}}} L^2_{\mathfrak{X}}([G])_{\mathfrak{C}},$$

where $\mathfrak{B}_{H-\mathrm{dist}}$ is defined in terms of the vanishing subspaces of the intertwining periods.

The technical core of the work is the study of the H-period of a pseudo-Eisenstein series of G. Let $P = M \ltimes U$ be a standard parabolic subgroup of G. A pseudo-Eisenstein series is defined by a sum (convergent for large R)

$$\theta_\phi(g) = \sum_{\gamma\in P\backslash G} \phi(\gamma g),$$

where ϕ lies in a space of equivariant functions $C_R(\mathbf{U}(\mathbb{A})M\backslash\mathbf{G}(\mathbb{A}))$ with a certain "rapidly decreasing" condition indexed by a positive real number R. The period integral admits a decomposition with respect to double cosets $P\backslash G/H$

$$\int_{[H]} \theta_\phi(h) = \sum_{x\in P\backslash X} I_x(\phi)$$

Here $X = \{g \in G : g\bar{g} = e\}$ is a set of representatives of the cosets G/H, and

$$I_x(\phi) = \int_{P_x\backslash G_x(\mathbb{A})} \phi(h\eta_x)dh.$$

In the above integral decomposition, we first showed only those M-*admissible* orbits in $P\backslash X$ contribute to the summation. These orbits are bijective to the M-orbits $M\backslash N_G(M)\cap X$. We further showed only the M-cuspidal orbits in $M\backslash N_G(M)\cap X$ contribute to the summation. To sum up, we obtained a shrinking index for summation:

$$(M\backslash N_G(M)\cap X)_{M-\mathrm{cusp}} \subseteq M\backslash N_G(M)\cap X \cong (P\backslash X)_{P-\mathrm{adm}} \subseteq P\backslash X.$$

Moreover, we define the *intertwining period*.

$$J(\theta_f^M, x, \lambda) = \int_{A_M^{M_x} U_x(\mathbb{A})M_x\backslash G_x(\mathbb{A})} (\theta_f^M)_\lambda(h\eta)dh.$$

Assuming the convergence of the intertwining period, we can further obtain the formula, our main result, as follows

$$\int_{[H]} \theta_\phi(h)dh = \sum_x \int_{\lambda_x + i(\mathfrak{a}_M^*)_x^-} J(\phi[\lambda], x, \lambda)d\lambda,$$

where x runs over the finite set of M-cuspidal orbits in $M\backslash N_G(M)\cap X$.

In order to prove the convergence, we prepared the orbit analysis. In particular, the parabolic subgroup $L(x)$ behaves well in the M-minimal case, and the other cases are related to it via a directed graph. Consequently, we are able to prove the convergence directly for the M-minimal case, while other cases can be reduced to the M-minimal cases.

5. Current Projects and Future Work

At present, I am pursuing the following projects and developing several related ideas:

- In collaboration with Pan Yan, we are establishing the fundamental properties for generic representations of GSpin $\times$ GL over local fields. In particular, we will obtain its coincidence with the gamma factor defined by the Langlands–Shahidi method.
- For odd unitary groups, local gamma factors have been studied in [CW25], where in particular, the equality between the Rankin–Selberg and Shahidi gamma factors was proved. The explicit computation of the Rankin–Selberg gamma factor for simple supercuspidal representations in this setting, together with the GSpin case mentioned above, is within reach.
- The recent work of Adrian, Henniart, Kaplan, and Oi [AHKO25] analyzed L-parameters of orthogonal groups via gamma factors, providing a conceptual framework for such studies. Their approach can likely be extended to further cases, such as GSpin and unitary groups.
- Constructing the Sp_{2n}-distinguished discrete spectrum of U_{2n} arises naturally as a continuation of [WX24].

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